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MODAL ANALYSIS OF NONPRISMATIC BEAMS - UNIFORM SEGMENT METHOD

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A method to determine the free vibration frequencies and mode shapes of non-prismatic beams that are end-mounted on various supports is described. The term Uniform Segment Method (USM) is used to distinguish this method from finite element (FEM) techniques. The analytical details are presented along with a description of the implementing computer routines. The results are compared with finite element models. Directions for continued research using this modelling technique are also outlined. Keywords: Beams (Structural)

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INTRODUCTION

Modal techniques are frequently employed to solve the free vibrational motion of beams. For uniform beams, the method is analytical and is described in the textbook literature (refs 1-4). However, when the beam's cross section is nonuniform, it is often difficult, if not impossible, to find an analytical solution. One of the most commonly used methods for beams in this category is to divide the beam into a finite number of uniform sections and solve the equations within each section. Klein (ref 5) developed a "component modes" method based on this concept which she used to analyze a cantilevered helicopter blade with a high degree of nonuniformity. For each section of the blade, modal displacements were expressed by a Rayleigh-Ritz expansion, and continuity between sections was enforced by Lagrange multipliers. Mittendorf and Greif (ref 6) used a similar approach for a beam with general boundary conditions. The mode shapes were expanded in a Fourier series, while the Lagrange multipliers were used to enforce the kinematic transfer across section boundaries. A finite element approach, such as the one developed by Resende and Doyle (ref 7), may also be used for these cases. They used a three-noded element and a formulation based upon the quadratic isoparametric class. Identical shape functions are used to define geometry, displacement, and variations in cross-sectional properties.

For beams with special geometrical properties, a number of analytical techniques may be used. Wang (ref 8) obtained a general solution for beams whose sectional properties vary as a power of the axial coordinate. Mode shapes were obtained in terms of generalized hypergeometric functions using the method of Frobenius. Wang (ref 9) used the Galerkin method to determine frequencies. Modes were represented by a set of Legendre polynomials which served as shape

functions throughout the spatial domain. Lau (ref 10) derived the frequency determinant for a profiled and truncated fixed-free beam with an attached end mass using Bessel functions of the second order. Goel (ref 11) investigated vibrations of linearly-tapered beams mounted upon rotationally-elastic boundaries.

All of these methods are satisfactory within the confines of their modelling approximations. For example, with finite elements, the mode shape between nodes is approximated by a piecewise set of cubic polynomials. Continuity at element boundaries is satisfied up to the second derivative, and in most applications this is all that is required. Unlike the FEM, analytical techniques use functional representations which are particularly useful in applications where differentiability is required.

The purpose of this work is to present a hybrid method (analytical/numerical) to model the free vibrations of statically-determinant beams on general supports. This method adapts the exact prismatic beam solution to nonprismatic cases. In addition to a presentation of this method, the computer routines that implement the method are explained and a few examples are compared to the results generated by the FEM method.

ANALYTICAL METHOD

The transverse motion of a general nonprismatic beam is described by the following partial differential equation (PDE) according to Thomson (ref 1):

$$(EIy'')'' + \frac{w}{g}\ddot{y} - \left(J + \frac{EIw}{gkAG}\right)\ddot{y}'' + \left(\frac{Jw}{gkAG}\right)\ddot{\ddot{y}} =$$

$$\sum_{n=1}^{N}\left[p_n(x,t,y,y',\ldots,\ddot{y})\right] + \frac{J}{kAG}\ddot{p}_n(x,t,y,y',\ldots,\ddot{y})$$

$$- \frac{EI}{kAG}p_n''(x,t,y,y',\ldots,\ddot{y}) - w \qquad (1)$$

where

E = Young's Modulus

I = transverse moment of inertia

J = pitch moment of inertia

G = modulus of elasticity in shear

A = area of beam's cross section

k = shear coefficient (function of cross section)

w = weight per unit length of beam

p_n = n-th forcing function (total of N)

g = acceleration of gravity

x = axial coordinate

t = time (independent variable)

y = transverse displacement (dependent variable)

$\cdot$ = time derivative

$'$ = space derivative

This equation takes into account all possible effects of beam dynamics. The second term accounts for the translational inertia of the beam, while the remaining terms model rotatory inertia and shear deformation effects. All terms are on a per length basis. The right side contains the driving loads which may be functions of the dependent variable as well as space and time.

The free vibration modes (or mode shapes) are determined by solving the homogeneous form of Eq. (1). For a prismatic structure (i.e., when I, E, J, G, A, w, k are constant throughout the spatial domain), the free vibration equation takes the form

$$EIy'''' + \frac{w}{g}\ddot{y} - \left(J + \frac{EIw}{gkAG}\right)\ddot{y}'' + \left(\frac{Jw}{gkAG}\right)\ddddot{y} = 0 \qquad (2)$$

The solution of this equation is straightforward and can be found in the works of Bozich (ref 12) and Dolph (ref 13). Following the method used by Bozich, the PDE becomes a function of the structural parameters and the natural vibration frequencies as follows:

$$EIy'''' + \left(J + \frac{EIw}{gkAG}\right)\omega^2 y'' - \frac{w}{g}\left(1 - \frac{J\omega^2}{kAG}\right)\omega^2 y = 0 \tag{3}$$

where

ω = natural vibration frequency

Equation (3) can be nondimensionalized by introducing a new independent variable $\bar{x}$ (=x/L). By normalizing each term with respect to w^2/g and using the parameters suggested by Kruszewski (ref 14), the resulting dimensionless equation, valid for any uniform beam, is

$$\frac{1}{K_B^2}\frac{d^4y}{d\bar{x}^4} + (K_R^2 + K_S^2)\frac{d^2y}{d\bar{x}^2} - (1 - K_B^2K_R^2K_S^2)y = 0 \tag{4}$$

where

$$K_B^2 = \omega^2 L^4 \frac{w}{gEI} \tag{5a}$$

$$K_S^2 = \frac{1}{L^2}\frac{EI}{kAG} \tag{5b}$$

$$K_R^2 = \frac{1}{L^2}\frac{Jg}{w} \tag{5c}$$

These coefficients represent the contributions of bending, shear, and rotatory inertia, respectively. In this form, Eq. (4) has the solution:

$$y(x,t) = [A\cosh\beta\bar{x} + B\sinh\beta\bar{x} + C\cos\alpha\bar{x} + D\sin\alpha\bar{x}]\cos\omega t \tag{6}$$

where

$$\alpha = K_B\sqrt{\frac{+(K_R^2+K_S^2) + \sqrt{(K_R^2-K_S^2)^2 + 4/K_B^2}}{2}} \tag{7a}$$

$$\beta = K_B \sqrt{\frac{-(K_R^2+K_S^2) + \sqrt{(K_R^2-K_S^2)^2 + 4/K_B^2}}{2}} \tag{7b}$$

The coefficients A, B, C, and D are determined by the boundary conditions of the beam. The vibration frequencies, ω, and their respective mode shape functions are found by applying the appropriate boundary conditions.

To adapt this analysis to the nonprismatic case, the beam is represented by a finite number of prismatic segments. Within each segment Eq. (6) portrays the mode shape for the assumed uniform properties of that segment. The conditions at the common boundary between adjacent segments must match to preserve continuity between segments. The appropriate end boundary conditions are applied at the extremities of the beam. The equation defining the mode shapes for each segment is developed from Eq. (6) as follows:

$$\phi_{ij}(\bar{x}) = A_{ij} \cosh \beta_{ij}\bar{x} + B_{ij} \sinh \beta_{ij}\bar{x} + C_{ij} \cos \alpha_{ij}\bar{x} + D_{ij} \sin \alpha_{ij}\bar{x} \tag{8}$$

where

i = segment number

j = mode shape number

A,B,C,D = function coefficients (by mode/segment)

Employing these segmented-mode shape functions, the natural vibration frequencies and mode shape coefficients can be determined to within an arbitrary constant for any set of end boundary conditions. For example, consider the case of a free-free beam which is composed of M segments. The boundary conditions are

$$\phi''_{1j}(0) = 0 \tag{9a}$$

$$\phi''_{Mj}(1) = 0 \tag{9b}$$

$$\phi'''_{1j}(0) = 0 \tag{9c}$$

$$\phi'''_{Mj}(1) = 0 \tag{9d}$$

Continuity at the common boundaries between the segments is preserved by equating displacement, slope, moment, and shear calculated by adjacent mode functions. These conditions are as follows:

$$\phi_{i-1j}(\bar{x}_i) = \phi_{ij}(\bar{x}_i) \tag{10a}$$

$$\phi'_{i-1j}(\bar{x}_i) = \phi'_{ij}(\bar{x}_i) \tag{10b}$$

$$(EI)_{i-1}\phi''_{i-1j}(\bar{x}_i) = (EI)_i\phi''_{ij}(\bar{x}_i) \tag{10c}$$

$$(EI)_{i-1}\phi'''_{i-1j}(\bar{x}_i) = (EI)_i\phi'''_{ij}(\bar{x}_i) \tag{10d}$$

where

$\bar{x}_i$ = left boundary of i-th segment

For a model containing M segments, there are 4M coefficients and one frequency to be evaluated at each mode shape. Setting a coefficient of the mode shape in the first segment to unity renders the system deterministic and yields unique solutions at each natural frequency.

A set of algebraic equations in terms of the natural frequency, ω_i, and the 4M-1 coefficients is presented in matrix form in Figure 1. The system matrix contains an orderly set of entries representing the enforcement of the boundary conditions (B.C.) and continuity conditions (C.C.). At global location 1,1 the 2 by 4 subarray contains the terms for evaluating the B.C. at $\bar{x} = 0$, whereas a similar array ending at N,N represents the B.C. at $\bar{x} = 1$. "Walking" along the main diagonal are M-1 subarrays, 4 by 8 in size, which represent the C.C. across segment boundaries. The global address of the upper left corner of the initial subarray is 3,1 and is incremented 4,4 for each additional interface. The remaining terms are zero. The nonzero entries are functions of the segment properties, end-point locations, and the vibration frequencies. Setting the determinant equal to zero and solving for the roots of the resulting characteristic equation produces values for these frequencies. At each frequency, a

reduced system is developed by setting the appropriate mode shape coefficient in the first segment to unity, eliminating the first row, and shifting the corresponding column to the right side of the matrix equation. This new system of equations is deterministic at each frequency. The values for the remaining mode shape coefficients may then be calculated. The orderliness of the entries to the system matrix enables an efficient and dedicated computer code for its solution to be developed.

This method has one main advantage over finite element techniques because the segmenting of naturally uniform sections is not required since the mode function is exact within this domain. Thus, only a few segments are needed for structures with uniform or gradually varying sections. Another advantage of this method is that the number of modes which can be calculated is not a function of the number of segments. In finite elements, the mode shapes are defined by the location of the model's nodes so that an accurate representation of the mode shapes at higher frequencies requires a large number of nodes. This is not the case for the USM, since its "nodeless" elements use trigonometric and hyperbolic functions to represent the mode shapes. Finally, in cases where the modes are used in dynamic analyses (ref 15) and the loads are functions of the dependent variable and its derivatives, the USM mode functions can accurately represent these loads since they are differentiable. The orthogonality of the USM-generated mode shapes permits the use of the usual expansion techniques. Their finite element counterparts are usually represented by cubic polynomials which possess limited differentiability.

COMPUTER CODES

A set of FORTRAN-77 computer routines has been written and tested on an IBM 4341-N12 minicomputer driven by the VM/SP Operating System. A total of 2000

lines of code was written for this analysis. The overall operation is controlled by an executive routine called MOD-MAIN. This routine directly calls 7 of the 20 subroutines used in the analysis. Figure 2 is a flowchart of the overall operation. The data entry and initialization portion of the analysis is shown on the left side of the figure. Specifically, input data consists of the physical and geometric characteristics of the beam, the location of segment borders, and the boundary conditions at the beam extremities. In addition, the number of modes to be determined, parameters regarding the search procedure, tolerance on the root estimates, and output requirements are read from a stored disk file.

The right section of Figure 2 shows the flow through the analytical portion of the routines. A loop is shown that is traversed once for each mode requested. During each pass, a vibration frequency and set of mode shape coefficients are determined and the results sent to various disk files. The procedure for locating a root is contained within the routine labelled ROOT SOLVER. In this solver, a search procedure is conducted by marching along the frequency axis in fixed steps, filling the system matrix, and calculating its determinant at each step. A change in the sign of the determinant between a pair of adjacent frequencies signifies the existence of a root between these frequencies. A modified secant method (ref 16) is then invoked to accelerate the search procedure. The root is determined to within the specified tolerance and is passed back to MOD-MAIN along with current values in the system matrix. The mode shape coefficients are then determined by a call to the coefficient solver. In these routines the system matrix is reduced according to the boundary conditions imposed on the left end and the coefficient used for normalization. The new set of linear equations is then solved for the unknown mode shape coefficients and control is passed back to the MOD-MAIN. The output is generated by calls to the

various output routines. The program then returns to the top of the loop, the current frequency is incremented, and the process is repeated for the next root. The looping continues until all of the requested roots and coefficients are found, and the associated output is generated.

TEST CASES

This method was evaluated by comparing it to results generated by an FEM code. The software chosen was ABAQUS Release 4.7 developed by Hibbitt, Karlsson, and Sorenson (ref 17). The beam used in this study was 2540 millimeters (mm) long with a tapered circular cross section starting at a left end diameter of 125 mm. The diametral tapers ranged from 0.005 to 0.020.

A sketch of the beam and its two modelling representations is shown in Figure 3. The finite element model consists of 53 nodes and 52 two-noded linear elements. The beam elements consider the effect of shear but not rotatory inertia. The tapered section was subdivided into 13 prismatic sections with four elements in each section.

The USM model contains three segments of equal lengths whose diameter is the average diameter of that section of the beam. Six boundary condition combinations out of a possible 32 were evaluated for four diametral tapers. The first five bending frequencies were calculated yielding a total of 120 "data" points for each model. The entire 120 runs were accomplished in one afternoon.

A comparison between each pair of values indicates that at most a 6.7 percent discrepancy exists between the results of both models. Most calculated values were well within this extreme. In general, the FEM values were greater than those obtained from the USM for the first three modes. However, the reverse is true for modes four and five. These comparisons are shown graphically in Figures 4 through 6 for three of the boundary conditions. In these

graphs the frequency is plotted against diametral taper for all modes. The upright triangles represent the values determined by the USM, whereas the inverted triangles are results calculated by the FEM. Overall results indicate that the effect of the taper is greater for the higher modes. For example, for the fix-pin beam, the fifth frequency is reduced from 970 to 820 Hz as the taper is increased from 0.005 to 0.020, a 15 percent reduction. The primary frequency, however, has been reduced by only 8 percent from 60 to 55 Hz.

In Figures 7 through 9, the fourth bending mode shape of each model is compared for the extreme taper value (0.020). The shapes are very similar, but the magnitudes of the USM displacements are slightly greater than those from the FEM. The greatest discrepancy of 11 percent occurs for the fix-guide boundary condition at the three-quarter point along the axis. In retrospect, the USM model could have been refined by either changing the geometric properties of each segment or by adding more segments to better approximate the taper.

To compare the differences in mode shapes as a function of taper, consider the results plotted in Figure 10. This figure contains the fourth mode shape generated by the USM for the fix-pin boundary condition at all values of the taper. As is expected, the beam possessing the thinnest cross section at the right boundary has the greatest deflection. The deflection for the thinnest beam is about 15 percent greater than for the heaviest beam. A frequency shift from 650 to 550 Hz is also indicated.

To compare the differences between beam modelling PDE, consider the results shown in Figure 11. In this figure, the mode shapes for the fourth bend mode and the steepest taper are plotted for all three models using the USM. The boundary conditions are fix-pin. The difference between the shapes is hardly discernible for this geometry. The frequency shift is about 20 Hz, the highest

being associated with the Euler model. This type of response is expected since inclusion of additional terms in the PDE tends to retard all flexural free vibration frequencies.

DISCUSSION AND CONCLUSIONS

This work, which was developed in conjunction with the author's dissertation (ref 15), presents an accurate and efficient method to determine the free vibration frequencies and mode shapes of nonprismatic end-mounted beams on standard boundaries. The term Uniform Segment Method has been coined to distinguish this modelling from the more popular finite element techniques. A dedicated computer routine has been developed and tested against results obtained from an FEM model for a tapered beam. The maximum discrepancy for any frequency calculation was 7 percent, whereas the maximum difference in any local value of the mode shape was 11 percent. Greater accuracy can be obtained by a modest increase in the number of USM segments.

Future work with this method includes linking the model to an optimization package so that discrepancies between USM-calculated frequencies and a set of known frequencies (either FEM-generated or experimentally determined) are minimized. The optimization variables would be the segmentation spacing and the section properties of the segments. Linearly elastic boundary conditions (in both transverse and rotational directions) will be included along with the standard boundary types. Finally, the development of a transient analysis package using the differentiable USM modes will permit the determination of forced responses for a variety of displacement-dependent loadings.

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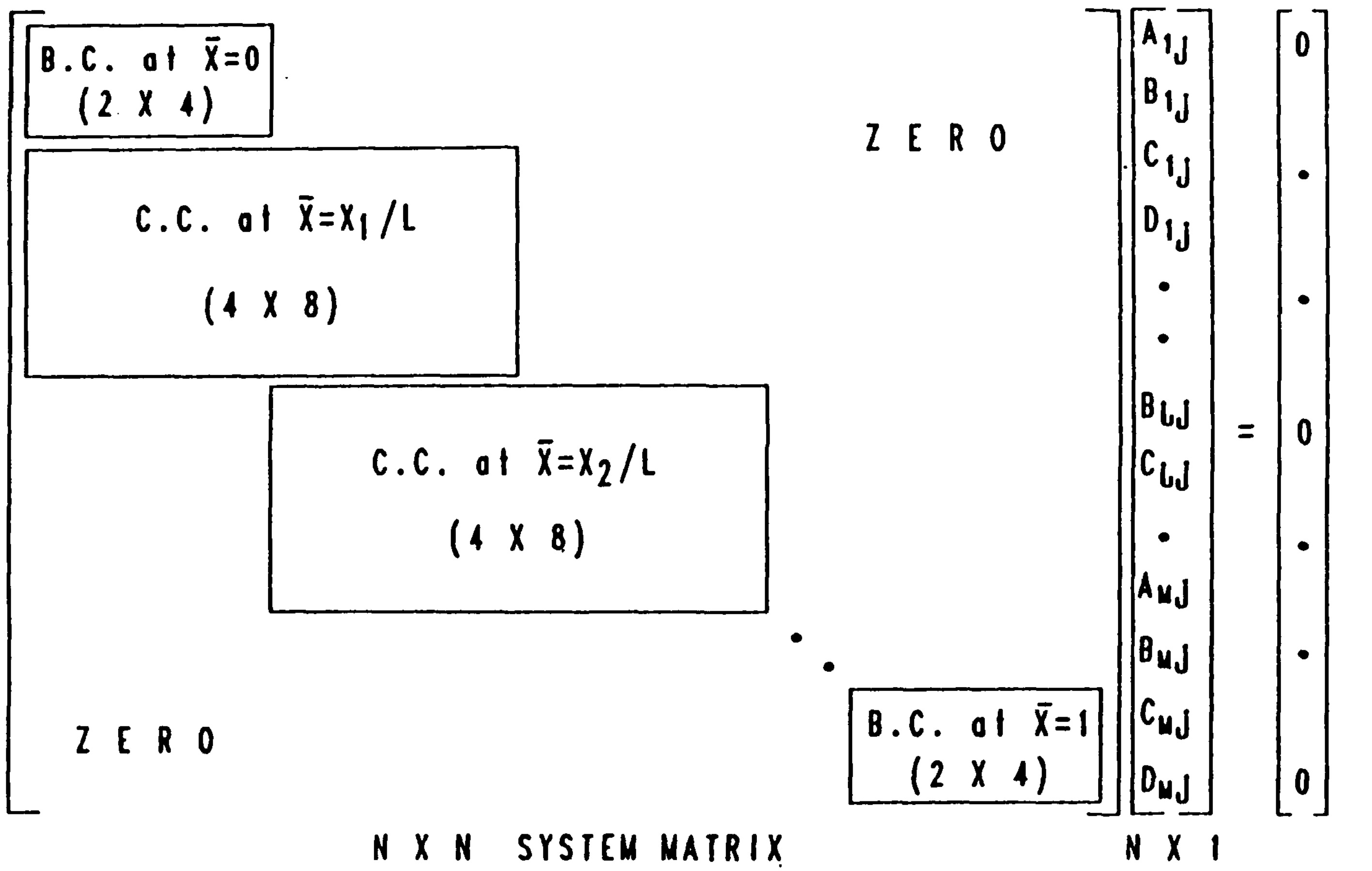
UNIFORM SEGMENT METHOD: SYSTEM OF EQUATIONS



ι – SEGMENT NUMBER
j – MODE NUMBER

FIGURE 1. UNIFORM SEGMENT METHOD: SYSTEM OF EQUATIONS

UNIFORM SEGMENT METHOD: ANALYSIS FLOWCHART

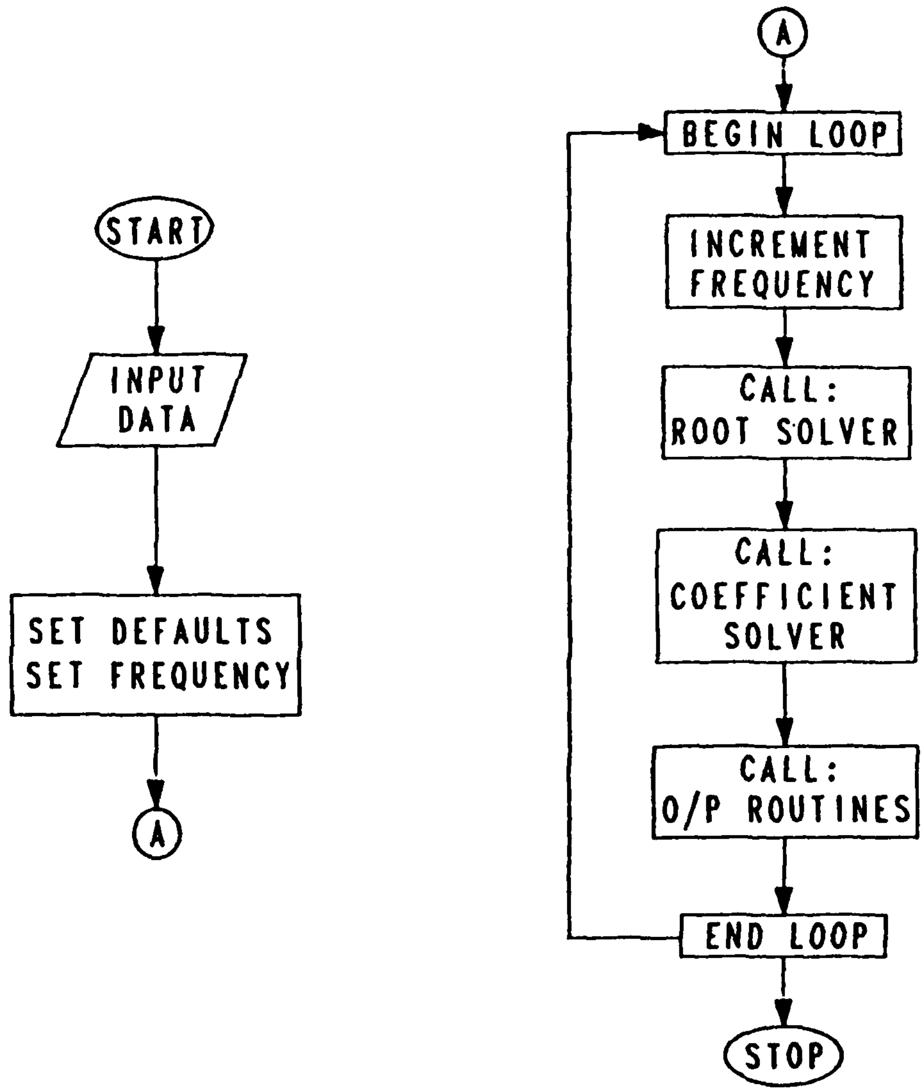


FIGURE 2. UNIFORM SEGMENT METHOD: ANALYSIS FLOWCHART

GEOMETRIES OF THE TAPERED BAR

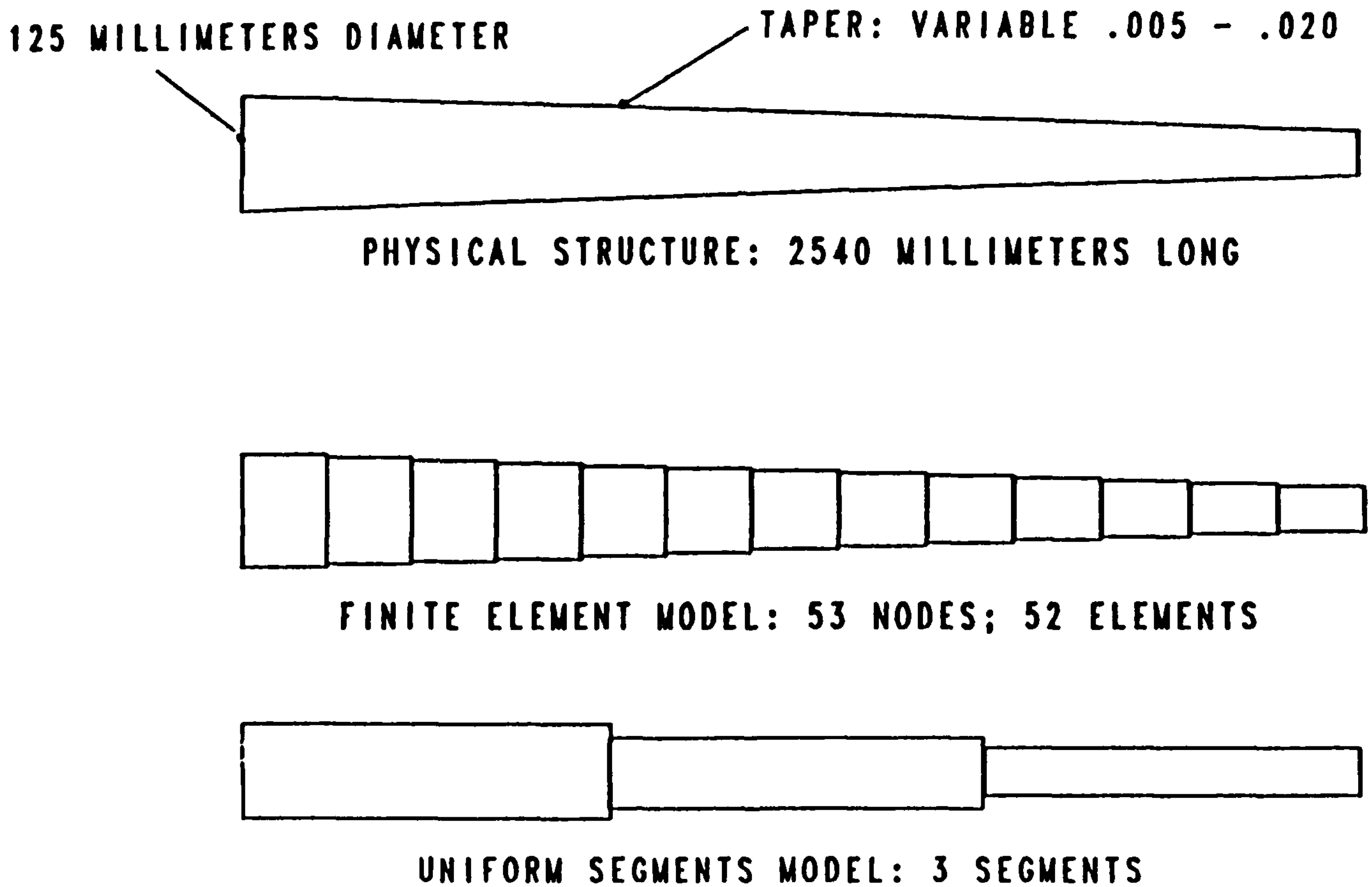


FIGURE 3. TEST CASES: GEOMETRIES OF THE TAPERED BAR

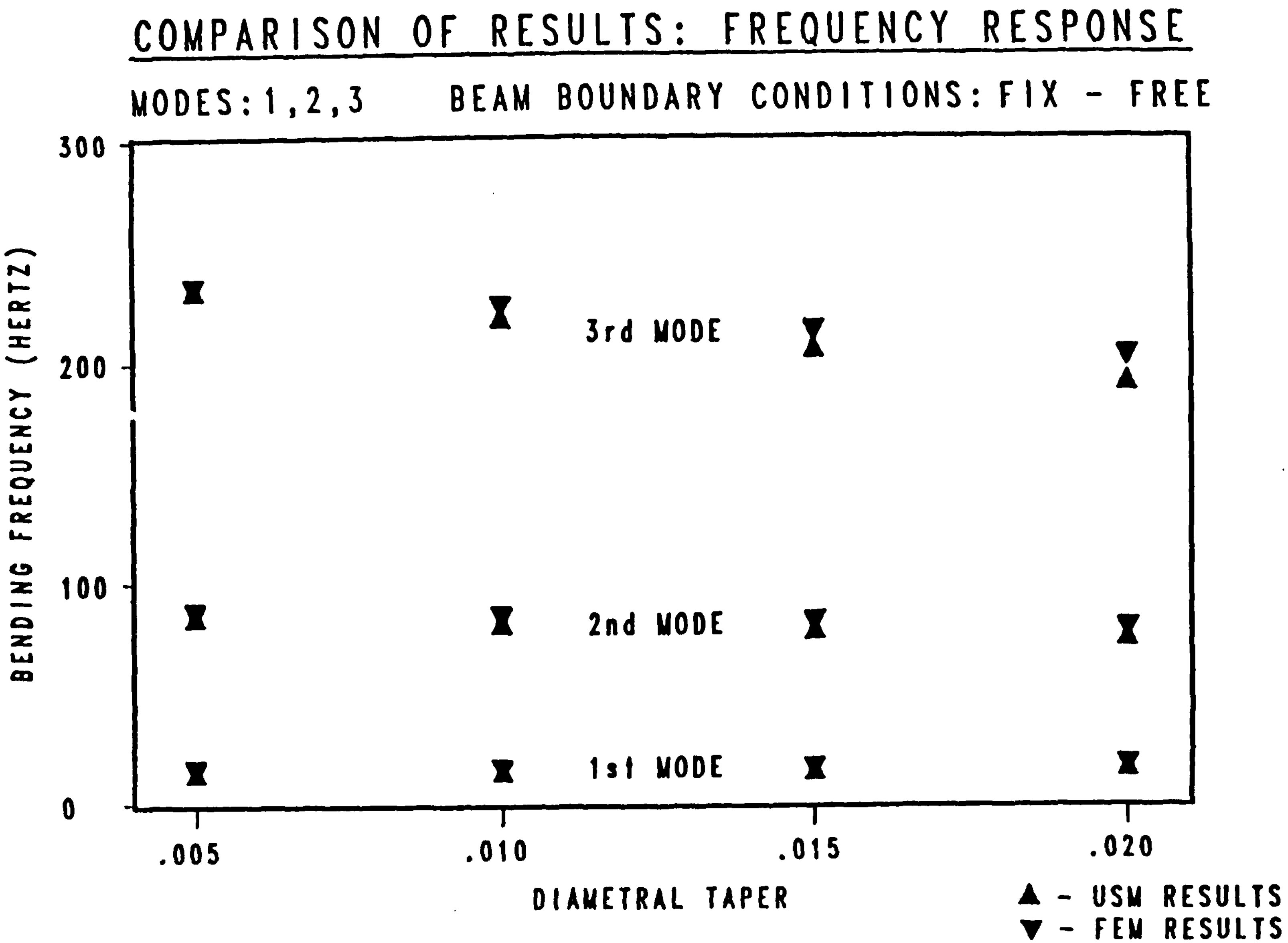


FIGURE 4A. FREQUENCY COMPARISON: MODES 1-3; FIX-FREE BEAM

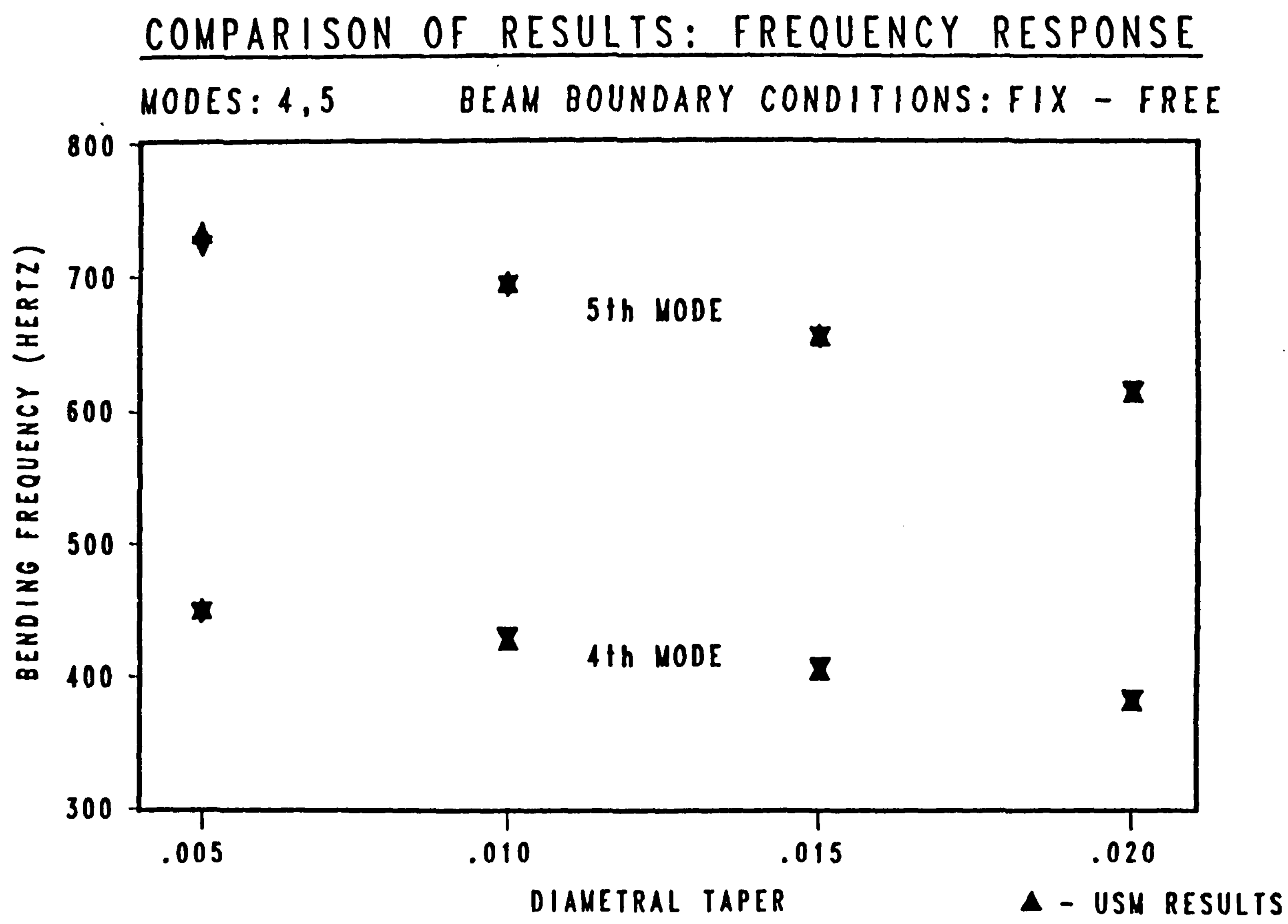


FIGURE 4B. FREQUENCY COMPARISON: MODES 4-5; FIX-FREE BEAM

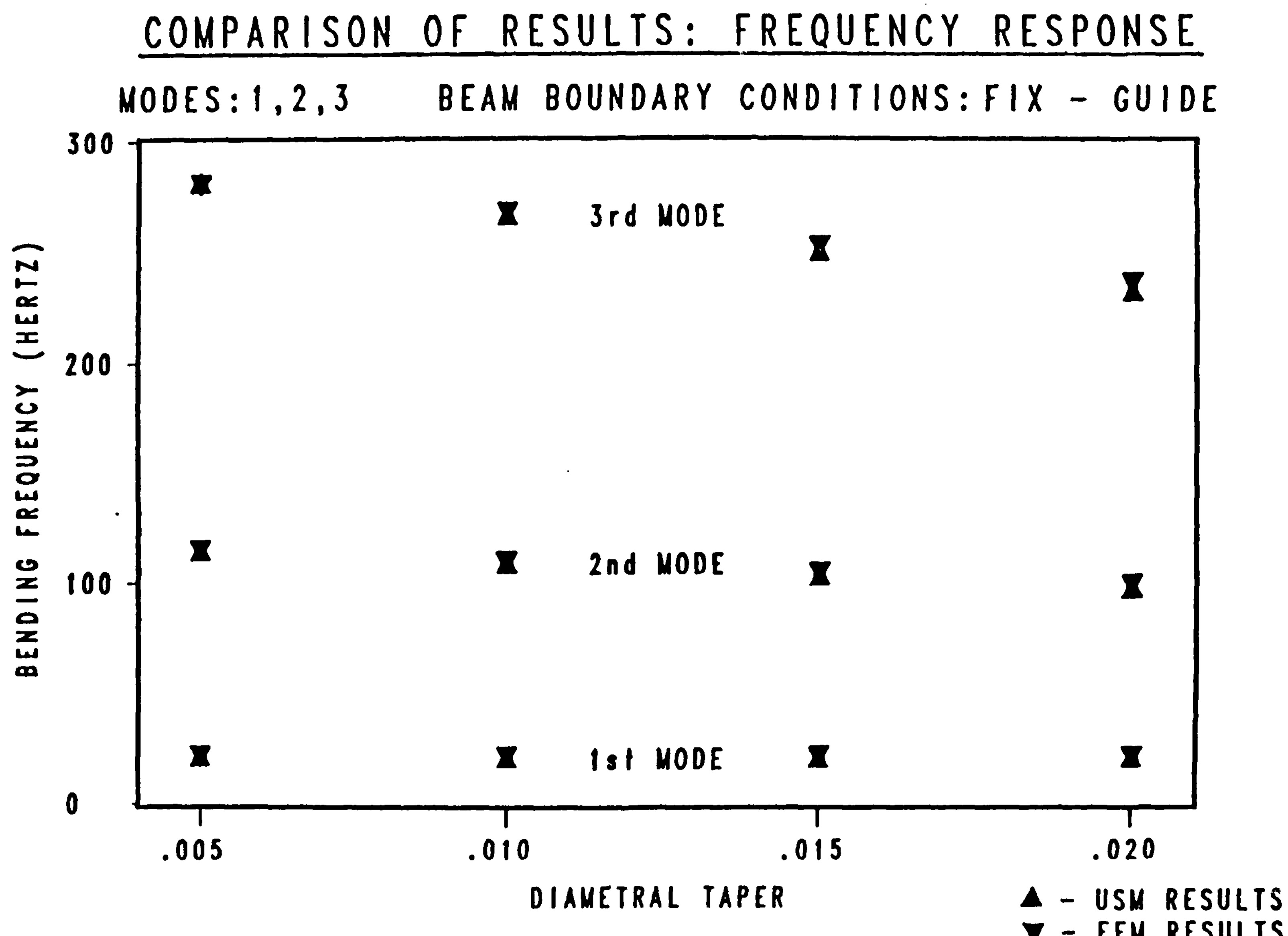


FIGURE 5A. FREQUENCY COMPARISON: MODES 1-3; FIX-GUIDE BEAM

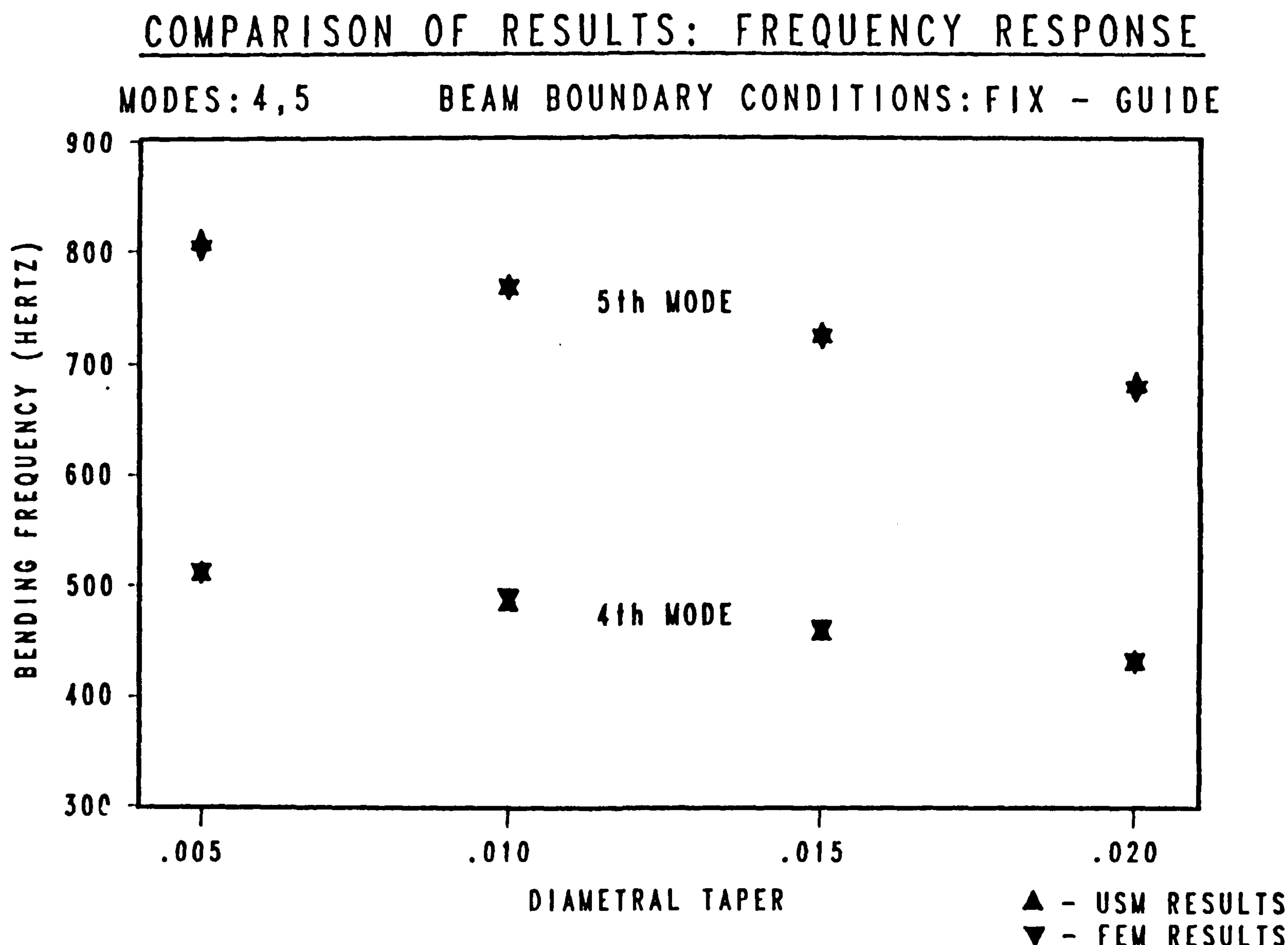


FIGURE 5B. FREQUENCY COMPARISON: MODES 4-5; FIX-GUIDE BEAM

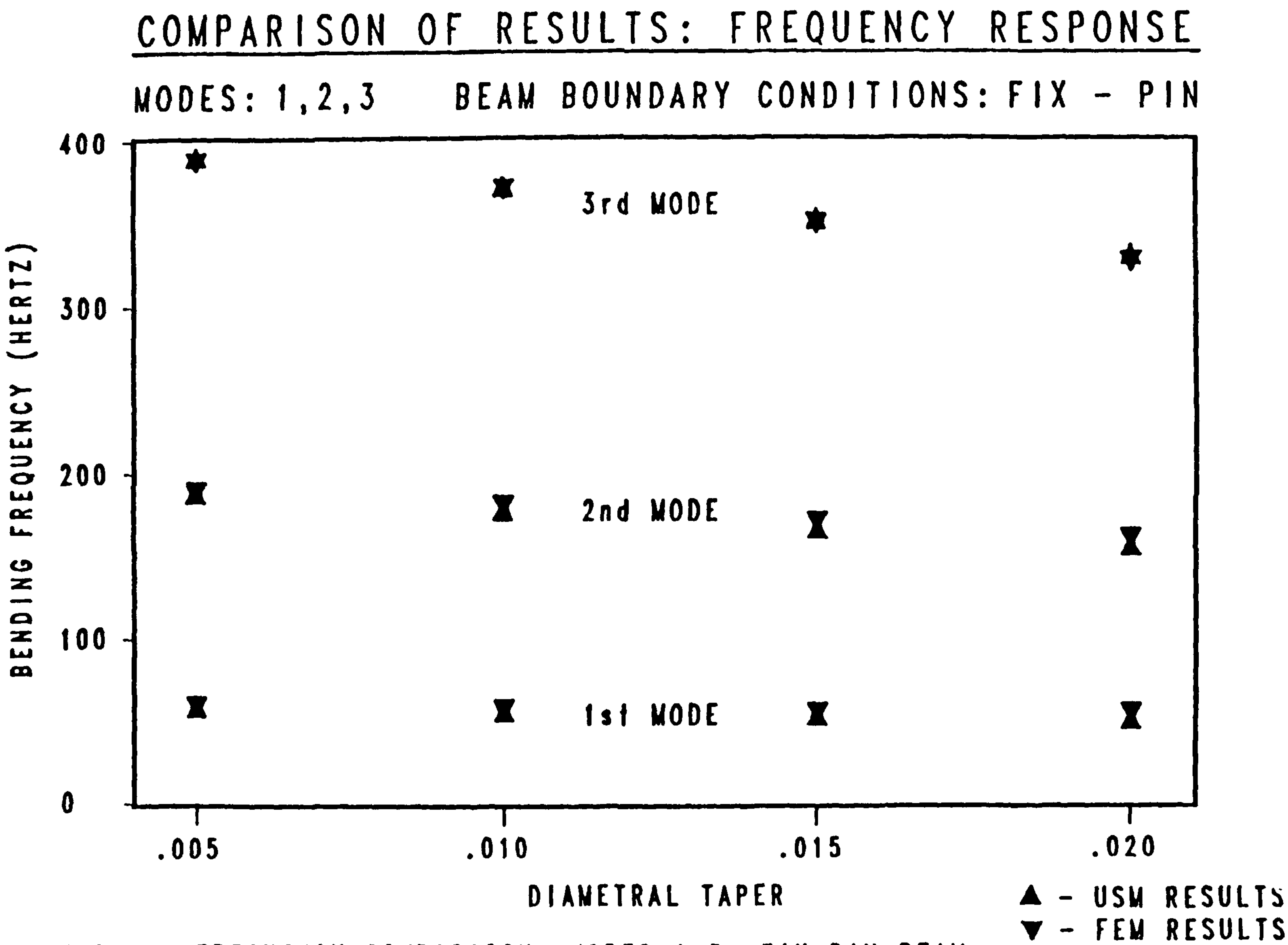


FIGURE 6A. FREQUENCY COMPARISON: MODES 1-3; FIX-PIN BEAM

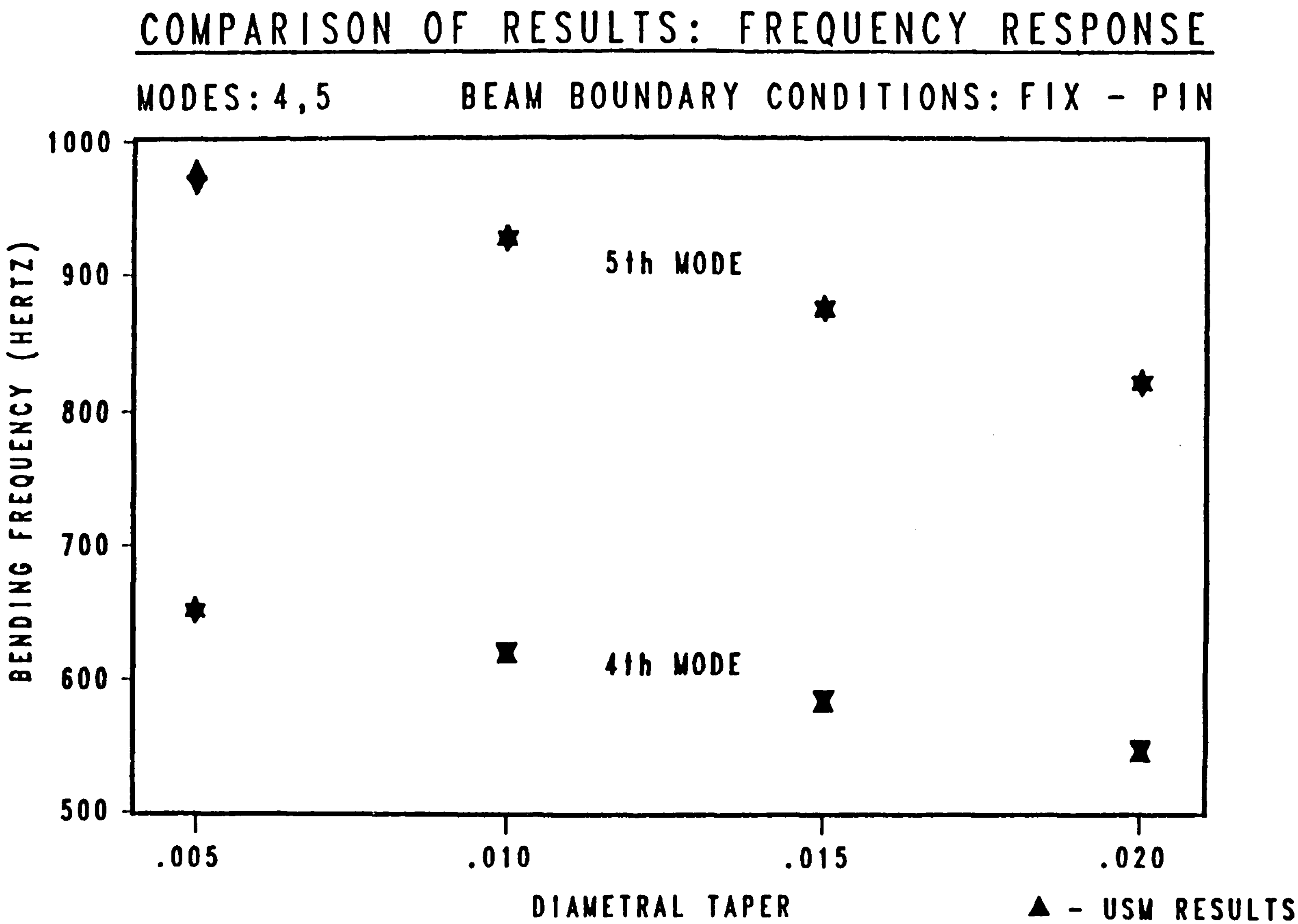


FIGURE 6B. FREQUENCY COMPARISON: MODES 4-5; FIX-PIN BEAM

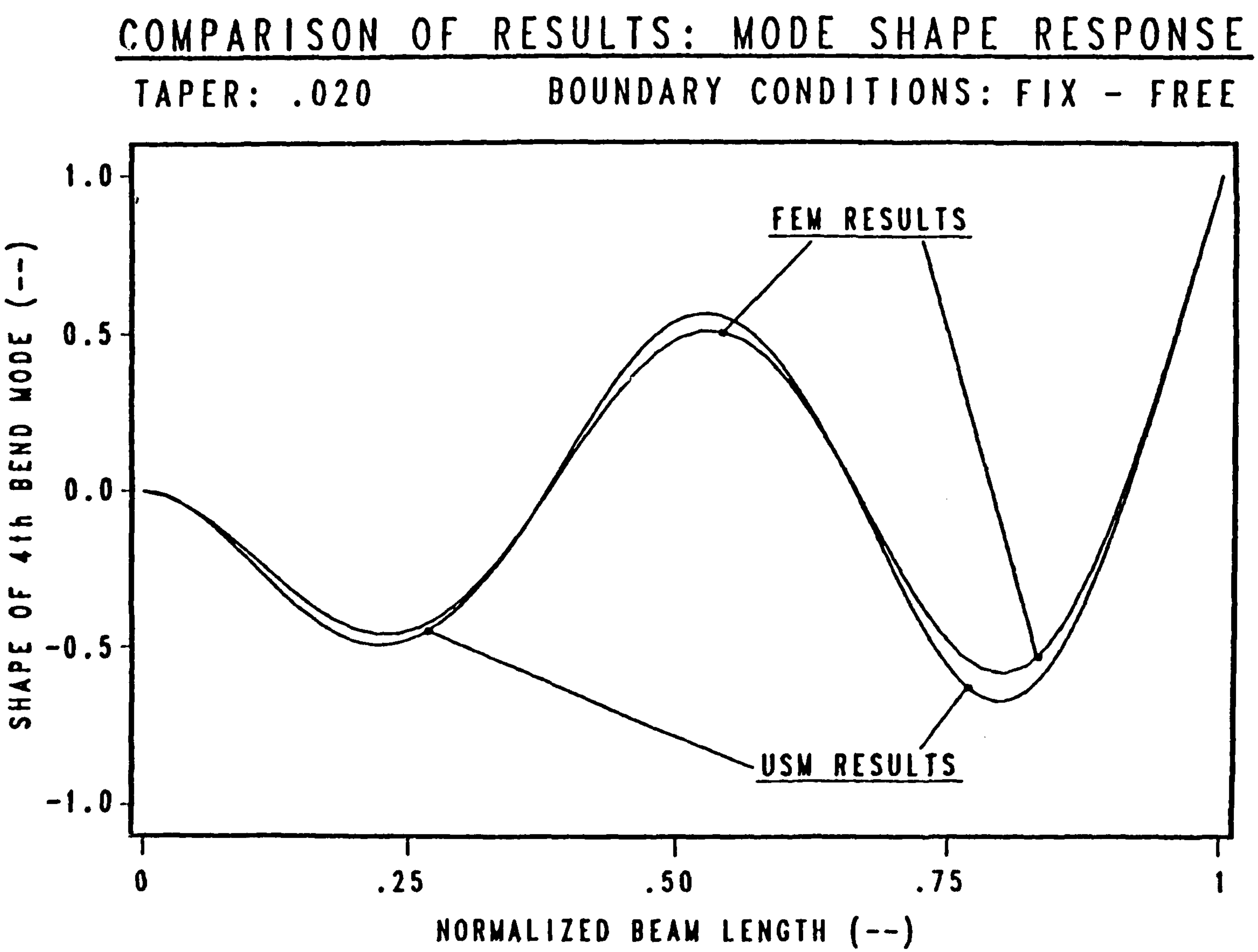


FIGURE 7. SHAPE COMPARISON: 4th MODE; FIX-FREE BEAM

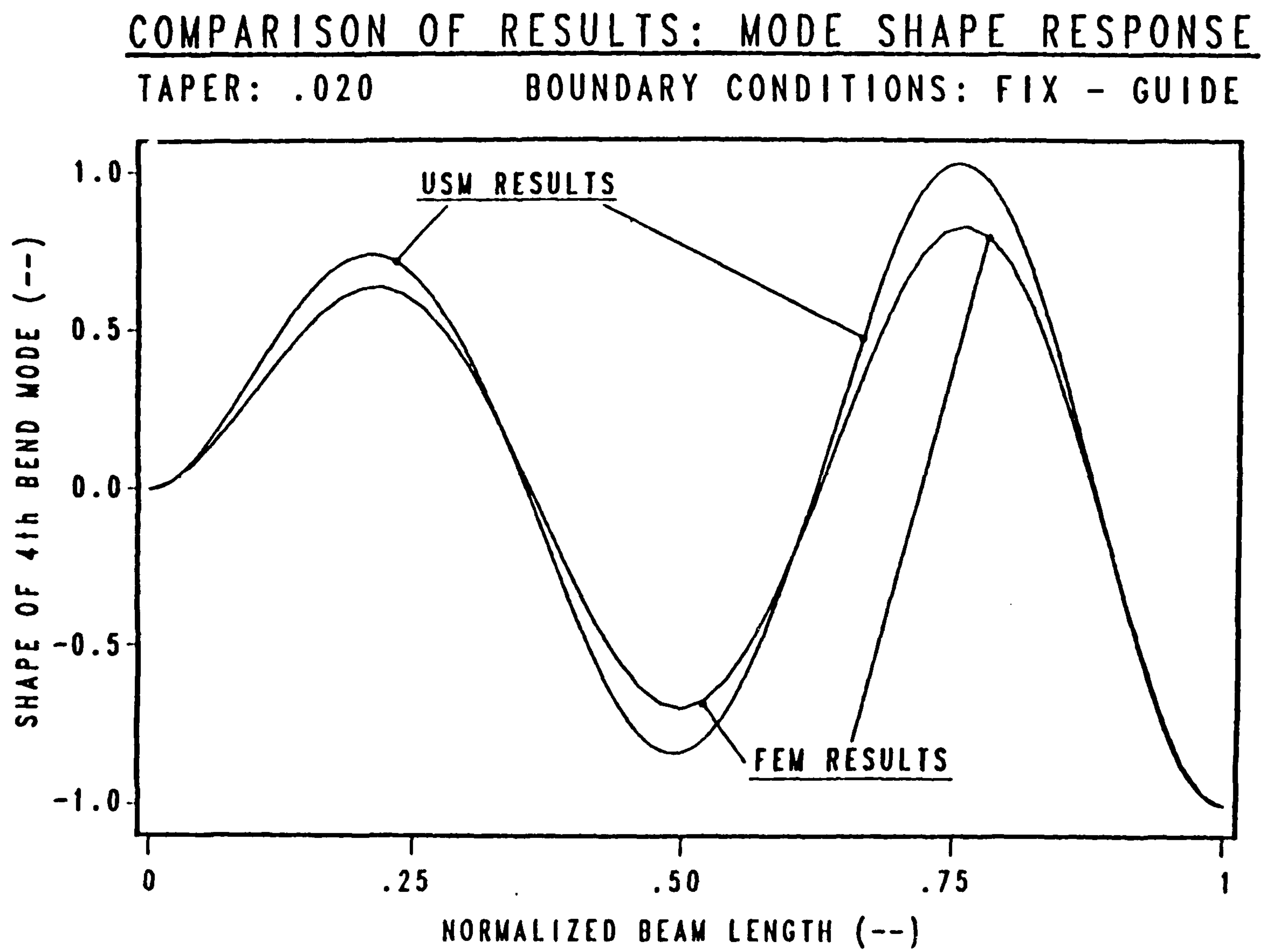


FIGURE 8. SHAPE COMPARISON: 4th MODE; FIX-GUIDE BEAM

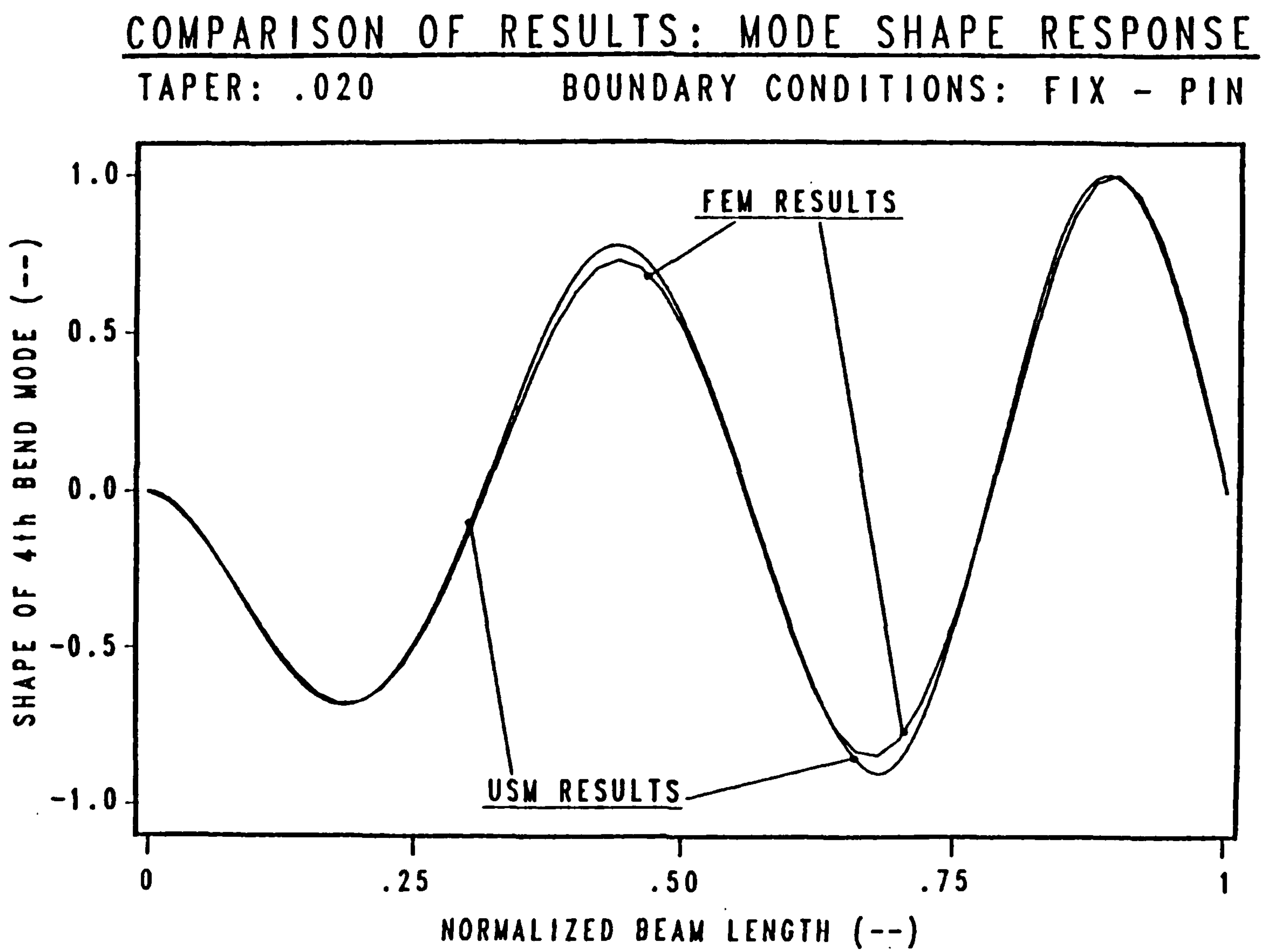


FIGURE 9. SHAPE COMPARISON: 4th MODE; FIX-PIN BEAM

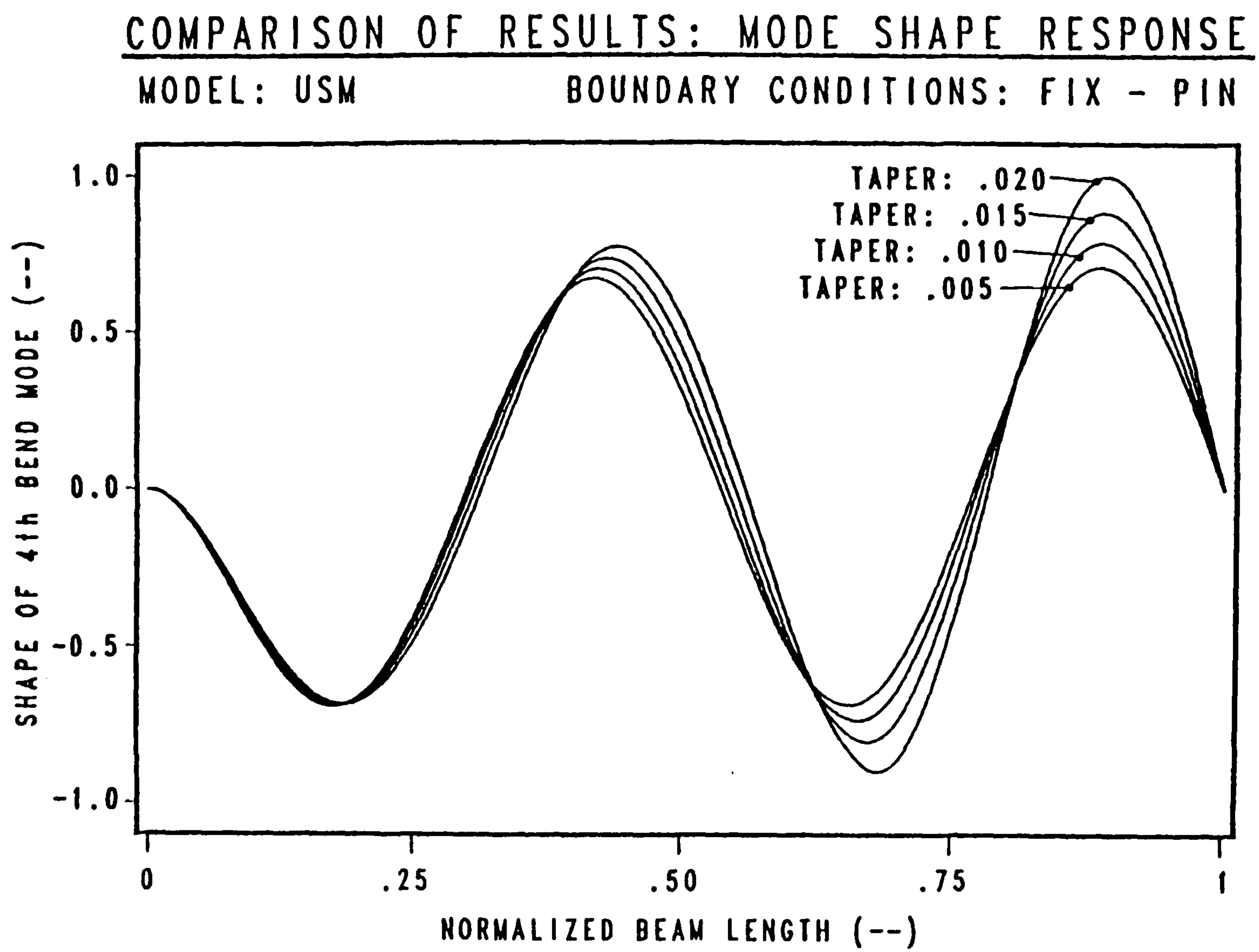


FIGURE 10. SHAPE COMPARISON VARIOUS TAPERS: 4th MODE; FIX-PIN BEAM

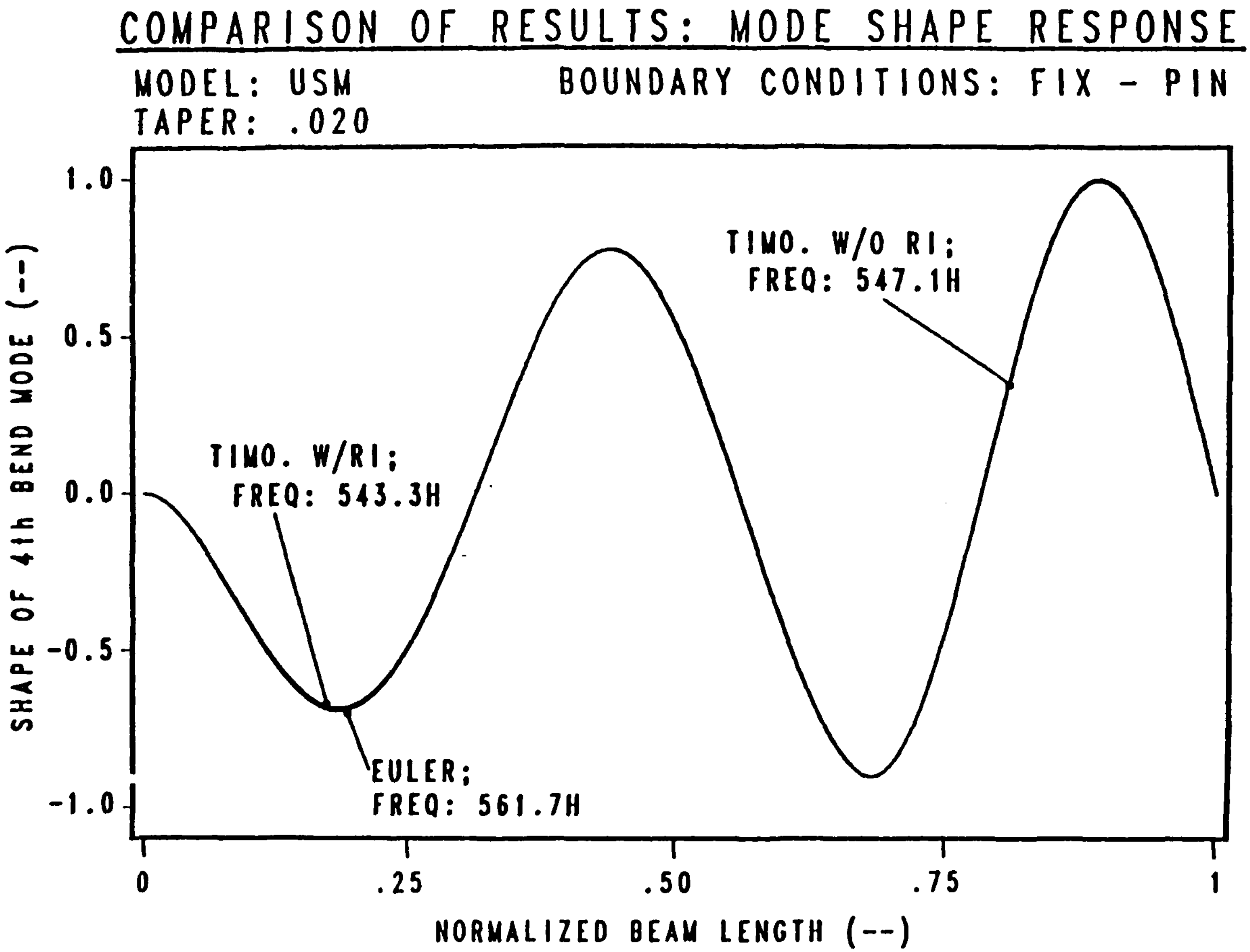


FIGURE 11. SHAPE COMPARISON VARIOUS pde: 4th MODE; FIX-PIN BEAM

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